**Classical Harmonic Oscillator**

In this assignment, we shall review some basic properties of classical harmonic oscillators, which are essentially to just about everything you’ll ever do in physics (be it classical or quantum physics).

1. Consider a particle of mass m, placed on a horizontal frictionless table, connected to a stationary wall by a spring with spring constant k. Assume that the unstretched spring has length x0. Draw this setup and identify the forces on the particle.
2. Write down the equation of motion for this particle (by equation of motion, I mean an equation depending only on x and its variables, in addition to m, k and any other parameter that you deem necessary). **In this and the remaining questions, mention if the unstretched length of the spring shows up in the answer. If so, why? If not, why not?**
3. Is there a way to get rid of x0 altogether by changing the reference point from which all positions are being measured? What is this new reference point?
4. Show that the equation of motion can be expressed as d2x/dt2=-w2x and find w in terms of the basic parameters m and k. Invert this to find the relation between k in terms of m and w, you’ll use it later.
5. Solve this equation of motion to find x(t). By “solve”, I don’t want you to write down the general solution out of nowhere. I want you to solve this differential equation stepwise, by integrating the given differential equation. **Hint**:Second order differential equations are hard; first order ones are easy. Can you think of a substitution to make the given equation a first-order equation?
6. How many integration constants should your solution above have? Does it have those many? If it doesn’t, then you screwed up somewhere.
7. Using common sense, think of the initial information needed to be able to fix the integration constants. Is this the only way to give the information? How would Cheshire cat do it? **Context:** The cat’s an asshole, so come up with a convoluted way of giving the required information. Also, show how you would find the required integration constants from the given information.
8. Assume that the particle starts at x=0. What constant of integration is fixed by this information? For x(0)=0, write down the expression for x(t) {it should depend on one integration constant, which we haven’t yet fixed}. From x(t), find v(t) and a(t) and verify that this expression for x(t) indeed satisfies the differential equation.
9. Find the velocity v as a function of the position x. What did you do get to this equation (express the strategy in words)? The point of this question is to remind you of something you learnt in high school coordinate geometry.
10. Using the expression for velocity as a function of x, identify when
    1. Value of x when v=0
    2. Value of v when x=0
    3. Value of x when v= half of its maximum value.
    4. Value of v when x= half of its maximum value
    5. What is your takeaway message from (c) and (d)?
11. Write down the kinetic energy of the particle as a function of x using what you have in the above problem. For what value of x is the kinetic energy maximum? For what value of x is it a minimum? Can the kinetic energy of a particle ever be negative? Explain why or why not.
12. In the above problem, you might have noticed that the kinetic energy is changing with x. Where is the kinetic energy going?
13. From the above observation, what can you say about energy conservation? In general, when do you expect it to hold for a system? Does the given system satisfy these requirements? Give an example of a system which does not satisfy these requirements.
14. From the observations made in the previous two problems, find the potential energy as a function of x. For what value of x is the potential energy maximum? For what value of x is it a minimum? Can the potential energy of this particle ever be negative? Explain why or why not. In general, can the potential ever be negative in any situation? If not, explain why. If yes, give an example.
15. Let us now try to find the potential energy in an alternate way.
    1. Write the expression for force F as a function of the position x.
    2. Find the work done by this force in **slowly** moving the particle from point xA to point xB.
    3. What is the relation between the above work and potential energy? (or potential energy change – identify the relevant quantity here).
    4. From the above relation, find the potential energy of the particle.
16. Plot the kinetic energy, the potential energy and the total energy of this particle as a function of x (do it in one plot). Describe the plots in words (is it a circle, a parabola, an oppai-shaped curve etc.).
17. Is the motion of this particle periodic? If yes, find the time-period.
18. Plot the kinetic energy, the potential energy and the total energy as a function of time. Explain the graphs in words and explain if they make sense.